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AN INFLUENCE ON EIGENVIBRATIONS IN RESONATORS OF ANISOTROPY OF BOUNDARY SURFACE OF PIEZOELECTRIC PLATE WITH VARIABLE CONVEXITY

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Abstract – **The report addresses an analysis of the transversely varying thickness modes in a piezoelectric resonator caused by the convex anisotropic boundary surface in a piezoelectric plate. We show that the anisotropy of the boundary surface greatly influences the frequency spectrum of the eigenvibrations of the resonator. The relations to calculate the frequency spectrum and the relative amplitudes of the vibrations are derived.**
Keywords – Flicker noise, spectral density, crystal oscillator, noise model

1. INTRODUCTION

An application of the anharmonic modes of crystal resonator as sensors of environment is challenging to get progress in self-contained frequency control in the precise type of crystal oscillators utilizing the modulational method [1]. The approach requires an accurate determination of the resonator frequency spectrum, sensor frequencies, and frequencies sensitivities to avoid the multi mode problems such as frequencies interaction and intermodulation. Modulation is by principle related to the nonlinear products which fall within the bandwidth of the sensor, and, as a result, both the unwanted splashes in the oscillator phase spectrum and the sensitivity irregularities may occur. Either phenomenon can readily be prevented at the early stage if to adjust slightly the electrode shapes and piezoelectric plate geometry based on a proper mathematical model of a resonator.

To exploit the resonator with sensors of environment, the mathematical model of a resonator must be developed and the vibrations spectrum must be investigated. The traditional crystal resonators with a spherical convex piezoelectric plate were studied in [2]—[4]. In the paper [5], the ellipsoidal geometry of the piezoelectric element was considered, where only the anisotropy in the direction of the normal to the plate is taken into account and in which the spherical symmetry is remained. In such models, the frequency spectrum remains almost unaltered. The only appreciable changes are fixed owing to the anisotropy of the piezoelectric constants in the direction of the plate normal.

In this paper the ellipsoidal model of the piezoelectric plate is examined taking into consideration the anisotropy 1) in the direction of the plate normal and 2) in the plate plane. Based upon, we show an essential influence of the boundary surface anisotropy on the frequency spectrum of eigenvibrations. Some new parameters of the piezoelectric resonator were introduced for the modes of eigenvibrations to optimize its performance.

2. EIGENVIBRATIONS OF A PIEZOELECTRIC RESONATOR WITH AN ELLIPSOIDAL PLATE

Let us examine the aforementioned crystal resonator (Fig. 1) with a circular convex piezoelectric plate. The relative radiuses in the direction of x, y and z are R_1, R_2 and R_3, respectively. The maximum thickness of the plate is depicted by L.

Following [2], the vibration frequency spectrum of a resonator is obtained by a solution of the effective differential equation in the principal axes x and y of the tensor $d_{\alpha\beta}$, that is

$$\begin{aligned}&-d_1\frac{\partial^2\Psi}{\partial x^2}-d_2\frac{\partial^2\Psi}{\partial y^2}\\&+\omega_{nF}^2\left(1+\frac{R_3}{L(R_1R_2)^2}\left[a_{11}x^2+a_{12}xy+a_{13}y^2\right]\right)\Psi(x,y),\\&=\omega^2\Psi(x,y)\end{aligned} \tag{1}$$

where d_1 and d_2 are eigenvalues of a tensor $d_{\alpha\beta}$ depending on the whole set of material constants for the piezoelectric crystal ([6]), ω_{nF} is the fundamental frequency of the *n*th overtone, and

$$\begin{aligned}a_{11}&=R_2^2\cos^2\alpha+R_1^2\sin^2\alpha,\\a_{12}&=\left(R_2^2-R_1^2\right)\sin 2\alpha,\\a_{13}&=R_2^2\sin^2\alpha+R_1^2\cos^2\alpha.\end{aligned} \tag{2}$$

We now substitute $x=\sqrt{d_1}\tilde{\xi}$ and $y=\sqrt{d_2}\tilde{\eta}$ and arrive at the equation :

$$\frac{\partial^2\Psi^{(0)}}{\partial\tilde{\xi}^2}+\frac{\partial^2\Psi^{(0)}}{\partial\tilde{\eta}^2}+\Big[\omega^2-\omega_{nF}^2-\frac{\omega_{nF}^2 R_3}{L(R_1R_2)^2}\left\{a_{11}d_1\tilde{\xi}^2+a_{12}\sqrt{d_1d_2}\,\tilde{\xi}\tilde{\eta}+a_{13}d_2\tilde{\eta}^2\right\}\Big]\Psi^{(0)}(\tilde{\xi},\tilde{\eta})\cdot=0 \tag{3}$$

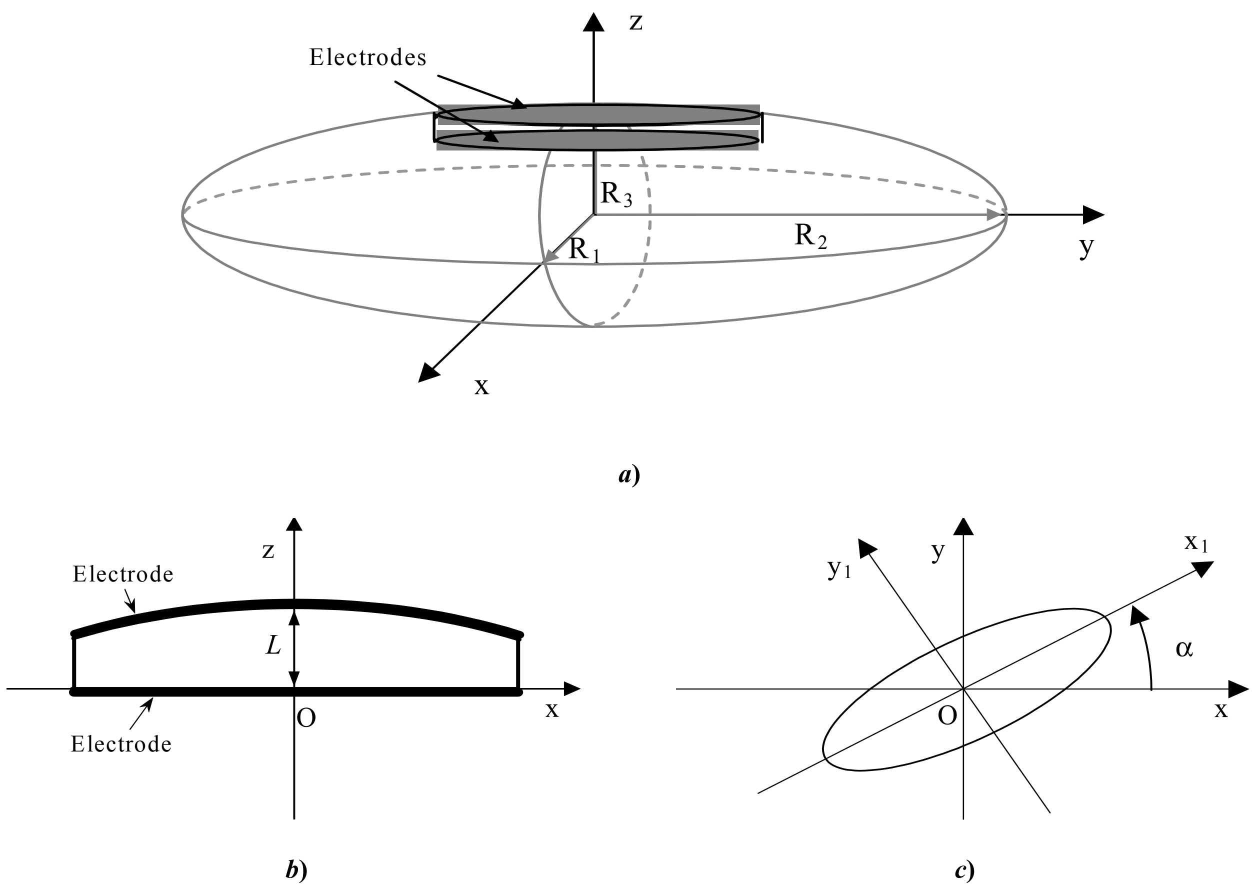

Figure 1. A model of a crystal resonator with a convex piezoelectric plate of a ellipsoidal shape, where x and y are self axes: *a*) basic concept, *b*) cross-section, *c*) a plan-view, α - the angle between the axes of the tensor $d_{\alpha\beta}$ and the ellipsoidal boundary surface.

We now rotate on an angle β to exclude the product of $\tilde{\xi}\tilde{\eta}$ in (3), introducing

$$\begin{cases}\tilde{\xi}=\xi\cos\beta+\eta\sin\beta\\ \tilde{\eta}=-\xi\sin\beta+\eta\cos\beta\end{cases}, \tag{4}$$

where

$$(a_{11}d_1-a_{13}d_2)\sin 2\beta+a_{12}\sqrt{d_1d_2}\cos 2\beta=0,$$
$$\beta=\frac{1}{2}\operatorname{arctg}\left(\frac{a_{12}\sqrt{d_1d_2}}{a_{13}d_2-a_{11}d_1}\right). \tag{5}$$

As a result of this manipulation, we obtain

$$\frac{\partial^2\Psi^{(0)}}{\partial\xi^2}+\frac{\partial^2\Psi^{(0)}}{\partial\eta^2}$$
$$+\left[\omega^2-\omega_{nF}^2-\frac{\omega_{nF}^2 R_3}{L(R_1R_2)^2}\left\{c_{11}\xi^2+c_{12}\eta^2\right\}\right]\Psi^{(0)}(\xi,\eta),$$
$$=0 \tag{6}$$

where

$$\left.\begin{aligned}c_{11}&=a_{11}d_1\cos^2\beta\\&-\frac{1}{2}a_{12}\sqrt{d_1d_2}\sin 2\beta+a_{13}d_2\sin^2\beta\end{aligned}\right\} \tag{7}$$

and

$$\left.\begin{aligned}c_{12}&=a_{11}d_1\sin^2\beta\\&+\frac{1}{2}a_{12}\sqrt{d_1d_2}\sin 2\beta+a_{13}d_2\cos^2\beta\end{aligned}\right\} \tag{8}$$

We now find a solution of the differential equation (6) in a form of $\Psi^{(0)}(\xi,\eta)=f(\xi)g(\eta)$ via

$$f''(\xi)g(\eta)+f(\xi)g''(\eta)$$
$$+\left[\omega^2-\omega_{nF}^2-\frac{\omega_{nF}^2 R_3}{L(R_1R_2)^2}\left\{c_{11}\xi^2+c_{12}\eta^2\right\}\right]f(\xi)g(\eta)$$
$$=0$$

and arrive at

$$\Psi^{(0)}(\xi,\eta)=\left(\frac{\sqrt{b_1}}{\pi}\right)^{1/4}\frac{1}{\sqrt{2^k k!}}\left(\frac{\sqrt{b_2}}{\pi}\right)^{1/4}\frac{1}{\sqrt{2^l l!}} \tag{9}$$
$$\times e^{-\frac{\sqrt{b_1}}{2}\xi^2}H_k(\xi b_1^{1/4})e^{-\frac{\sqrt{b_2}}{2}\eta^2}H_l(\eta b_2^{1/4})$$
$$\omega_{nkl}^2=\omega_{nF}^2+2\sqrt{b_1}\left(k+\frac{1}{2}\right)+2\sqrt{b_2}\left(l+\frac{1}{2}\right) \tag{10}$$

where $k,l=1,2,3,\ldots$,

$$b_1=\frac{\omega_{nF}^2 R_3}{L(R_1R_2)^2}c_{11},$$
$$b_2=\frac{\omega_{nF}^2 R_3}{L(R_1R_2)^2}c_{12}, \tag{11}$$

and $H_k(\cdot)$ are the Hermit polynomials.

Now we introduce the dimensionless anisotropy-dependent variables $\varepsilon=\frac{d_2}{d_1}$ and $\delta=\frac{R_2}{R_1}$, which involve properties of the piezoelectric material and boundary surface, respectively. Thereafter, the frequency spectrum of a piezoelectric plate is calculated by

$$\omega_{nkl}^2=\omega_{nF}^2$$
$$+2\omega_{nF}\sqrt{\frac{R_3d_1}{LR_2{}^2}}\left[\sqrt{c(\beta)}\left(k+\frac{1}{2}\right)+\sqrt{c\left(\beta+\frac{\pi}{2}\right)}\left(l+\frac{1}{2}\right)\right], \tag{12}$$

where $n=1,2,3,\ldots$, and the auxiliary functions

$$\left.\begin{aligned}c(\beta)&=\gamma(\alpha)\cos^2\beta\\&+\frac{\sqrt{\varepsilon}}{2}(1-\delta^2)\sin 2\alpha\sin 2\beta+\varepsilon\gamma\left(\alpha+\frac{\pi}{2}\right)\sin^2\beta\end{aligned}\right\} \tag{13}$$

and

$$\gamma(\alpha)=\sin^2\alpha+\delta^2\cos^2\alpha, \tag{14}$$

α - the angle between the axes of the tensor $d_{\alpha\beta}$ and the main axes of the ellipsoidal boundary surface.

Transforming (5) with (2), substituting from (14), we obtain

$$\beta=\frac{1}{2}\operatorname{arctg}\left(\frac{\sqrt{\varepsilon}(1-\delta^2)\sin 2\alpha}{\gamma(\alpha)-\varepsilon\gamma\left(\alpha+\frac{\pi}{2}\right)}\right). \tag{15}$$

Functions (13-15) describe the dependence of the eigenfrequencies ω_{nkl} on the anisotropy parameters ε and δ as well as on the mutual orientation of the axes of the piezoelectric plate and its boundary surface.

3. Conclusions

In this report, a new mathematical model of a piezoelectric resonator is presented of a piezoelectric plate of an ellipsoidal shape. The axes of ellipsoidal boundary surface are oriented arbitrary to the axes of the tensor $d_{\alpha\beta}$. In the particular case of coincidence of these axes such a problem was successfully solved in [7]. Considered model allows for accounting the additional parameters of a manufactured plate: angle α and the radius ratio $\delta=\frac{R_1}{R_2}$. We suppose using such a model to optimize performance of the piezoelectric resonator with the fundamental vibration mode and the anharmonic sensors of environment.

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